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# Big Bend Community College 

## Emporium Model <br> Math 99 Course <br> Workbook

A workbook to supplement
video lectures and online homework by:
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## Unit 13: <br> Compound Inequalities

To work through the unit, you should:

1. Watch a video, as you watch, fill out the workbook (top and example sections).
2. Complete Q1 and Q2 in WAMAP, put your work in the right column of the page.
3. Repeat \#1 and \#2 with each page until you reach the .
4. Complete the homework assignment on your own paper.
5. Repeat \#1 thru \#4 until you reach the end of the unit.
6. Complete the review/practice test on your own paper.
7. Take the unit exam.

### 13.1 Inequalities

## 13.1a Graphing

Inequalities:

- Less than:
- Less than or equal to:
- Greater than:
- Greater than or equal to:

Graphing on number line: Use $\qquad$ for less/greater than and use $\qquad$ when its "or equal to"

## Example 1:

$$
\text { Graph } x \geq-3
$$

## Example 2:

Give the inequality


Q1:

Q2:

## 13.1b Interval Notation

Interval notation:( , )
Use $\qquad$ for less/greater than and use $\qquad$ when its "or equal to" $\infty$ and $-\infty$ always use a $\qquad$

## Example 1:



Example 2:
Graph the interval $(-\infty,-1)$


Q1:

Q2:

Solving inequalities is very similar to solving $\qquad$ (with one exception...)

Three steps with inequalities: $\qquad$ then $\qquad$ then $\qquad$

Example 1:

$$
7+5 x \leq 17
$$

## Example 2:

$$
3(x+8)+2>5 x-20
$$

## 13.1d Multiply or Divide by a Negative

What happens to $5>-2$ when we multiply both sides by -3 ?
$(-3) 5 \ldots-2(-3)$

When $\qquad$ or $\qquad$ by a $\qquad$ you must

Three steps with inequalities: $\qquad$ then $\qquad$ then $\qquad$

Example 1:

$$
7-3 x \leq 16
$$

## Example 2:

$$
4<-2 x+16
$$

Q1:

Q2:

Tripartite inequalities:
When solving $\qquad$
When graphing $\qquad$
Three steps with inequalities: $\qquad$ , then $\qquad$ then $\qquad$

## Example 1:

$$
2 \leq 5 x+7<22
$$

## Example 2:

$$
5<5-4 x \leq 13
$$

Q1:

Q2:

### 13.2 Compound Inequalities

## 13.2a OR (two directions)

First, we will $\qquad$ each part above the number line, then we will $\qquad$ the union (OR) Symbol for Union:

Example 1:

$$
4 x+7<-5 \text { OR }-4 x-8 \leq-20
$$

Example 2:

$$
8 x+9<4 x-19 \text { OR } 2(4 x-8)-2 \leq 12 x-50
$$

Q1:

Q2:

## 13.2b OR (one direction)

With an OR if both graphs go the same direction than we use the

Example 1:

$$
4 x-6>10 \text { OR } 5-2 x \leq 7
$$

## Example 2:

$$
3 x+5<2 x-9 \text { OR } 7 x+3 \leq 5(x-1)
$$

## 13.2c AND (between)

AND:
First, we will $\qquad$ each part above the number line, then we will use the $\qquad$ (AND)

Example 1:

$$
6 x+5<11 \text { AND }-7 x+2 \leq 44
$$

## Example 2:

$11 x-10>3 x-2$ AND $2(5 x-3)+2 \geq 18 x-52$

Q1:

Q2:

## 13.2d AND (one direction)

With an AND if both graphs go the same direction than we use the $\qquad$

Example 1:

$$
5 x-6 \geq 26 \text { AND } 3 x+1>x-9
$$

## Example 2:

$$
2(4 x+4)>6 x+2 \text { AND } 7-x \leq 3+x
$$

## Q1:

Q2:

OR can give us $\qquad$ of number line or $\qquad$ in interval notation $\qquad$
AND can give us $\qquad$ of the number line or $\qquad$ in interval notation $\qquad$

## Example 1:

$$
2 x+1<x-3 \text { OR } 3(x+1) \geq x-15
$$

## Example 2:

Q1:

Q2:
$-3(4 x-1) \leq 15$ AND $2 x-3 \leq-9$

You have completed the videos for 13.2 Compound Inequalities. On your own paper, complete the homework assignment.

### 13.3 Absolute Value Equations

13.3a Two Solutions
$|x|=5$ so the $x$ could be $\qquad$ or $\qquad$
What is inside the absolute value can be $\qquad$ or $\qquad$
This means we have $\qquad$

## Example 1:

$$
|2 x-5|=7
$$

## Example 2:

$$
|7-5 x|=17
$$

## 13.3b Isolate the Absolute Value

Before we look at our two equations, we must first $\qquad$
Never $\qquad$ through absolute value!

Never $\qquad$ a term $\qquad$ an absolute value and a term $\qquad$ an absolute value!

## Example 1:

$$
5+2|3 x-4|=11
$$

## Example 2:

$$
-3-7|2-4 x|=-31
$$

Q1:

Q2:

With two absolutes, we need $\qquad$
The first equation is $\qquad$
The second equation is $\qquad$

## Example 1:

$$
|2 x-6|=|4 x+8|
$$

## Example 2:

$$
|3 x-5|=|7 x-2|
$$

Q1:
,


### 13.4 Absolute Value Inequalities <br> 13.4a GreatOR Than

$|x|>2$ means the $\qquad$ from zero is $\qquad$ than 2.


This is a graph of a compound $\qquad$ inequality. It can be written as $\qquad$
If the absolute value is greatOR than a number, we set up an $\qquad$

Example 1:

$$
|2 x-1| \geq 7
$$

Example 2:

$$
|7 x+4|>32
$$

Q1:

Q2:

## 13.4b Less Than

$|x|<2$ means the $\qquad$ from zero is $\qquad$ than 2.


This is a graph of a compound $\qquad$ inequality. It can be written as $\qquad$
If the absolute value is less than a number, we set up an $\qquad$

## Example 1:

$$
|3 x+7|<6
$$

## Example 2:

$$
|4 x+1| \leq 2
$$

Before setting up a compound inequality, we must first $\qquad$ the absolute value!

Beware: with absolute value we cannot $\qquad$ or $\qquad$

Example 1:

$$
2-7|3 x+4|<-19
$$

Example 2:

$$
5+2|4 x-1| \leq 17
$$

Q1:

Q2:

## Unit 14: Systems of Equations

To work through the unit, you should:

1. Watch a video, as you watch, fill out the workbook (top and example sections).
2. Complete Q1 and Q2 in WAMAP, put your work in the right column of the page.
3. Repeat \#1 and \#2 with each page until you reach the .
4. Complete the homework assignment on your own paper.
5. Repeat \#1 thru \#4 until you reach the end of the unit.
6. Complete the review/practice test on your own paper.
7. Take the unit exam.

### 14.1 Systems

14.1a Introduction to Substitution

Substitution: Replace the $\qquad$ with what it $\qquad$

Example 1:

$$
\begin{gathered}
x=-3 \\
2 x-3 y=12
\end{gathered}
$$

Example 2:

$$
\begin{gathered}
4 x-7 y=11 \\
y=-1
\end{gathered}
$$

Q1:

Q2:

## 14.1b Substitute an Expression

Just as we can replace a variable with a number, we can also replace it with an $\qquad$ Whenever we substitute it is important to remember $\qquad$

## Example 1:

## Example 2:

$$
\begin{gathered}
2 x-6 y=-24 \\
x=5 y-22
\end{gathered}
$$

Q1:
Q2:

To use substitution, we may have to $\qquad$ a lone variable

If there are several lone variables $\qquad$

## Example 1:

$$
\begin{gathered}
6 x+4 y=-14 \\
x-2 y=-13
\end{gathered}
$$

## Example 2:

$$
\begin{gathered}
-5 x+y=-17 \\
7 x+8 y=5
\end{gathered}
$$

Q1:
Q2:

## 14.1d Substitution Special Cases

If the variables subtract out to zero then it means either there is or $\qquad$

## Example 1:

$$
\begin{gathered}
x+4 y=-7 \\
21+3 x=-12 y
\end{gathered}
$$

## Example 2:

$$
\begin{gathered}
5 x+y=3 \\
8-3 y=15 x
\end{gathered}
$$

## 14.1e Addition/Elimination

If there is no lone variable, it may be better to use $\qquad$
This method works by adding the $\qquad$ and $\qquad$ sides of the equations together

## Example 1:

## Example 2:

$$
\begin{gathered}
-8 x-3 y=-12 \\
2 x+3 y=-6
\end{gathered}
$$

## Q1:

Q2:

$$
-5 x+9 y=29
$$

$$
5 x-6 y=-11
$$

## 14.1f Addition/Elimination and Multiplying an Equation

Addition only works if one of the variables have $\qquad$
To get opposites we can multiply $\qquad$ of an equation to get the value we want

Be sure when multiplying to have a $\qquad$ in front of either the $\qquad$ or the $\qquad$

## Example 1:

$$
\begin{gathered}
2 x-4 y=-4 \\
4 x+5 y=-21
\end{gathered}
$$

## Example 2:

$$
\begin{aligned}
& -5 x+3 y=-3 \\
& -7 x+12 y=14
\end{aligned}
$$

Q1:

## 14.1g Addition/Elimination and Multiplying Both Equations

Sometimes we may have to multiply $\qquad$ by something to get opposites

The opposite we look for is the $\qquad$ of both coefficients

## Example 1:

$$
\begin{aligned}
& -6 x+4 y=26 \\
& 4 x-7 y=-13
\end{aligned}
$$

## Example 2:

$$
\begin{gathered}
3 x+7 y=2 \\
10 x+5 y=-30
\end{gathered}
$$

## 14.1h Addition/Elimination Special Cases

If the variables subtract out to zero than it means either there is
or $\qquad$

## Example 1:

$$
\begin{aligned}
& 2 x-4 y=16 \\
& 3 x-6 y=20
\end{aligned}
$$

## Example 2:

$$
\begin{aligned}
& -10 x+4 y=-6 \\
& 25 x-10 y=15
\end{aligned}
$$

Q1:
Q2:

You have completed the videos for 14.1 Systems. On your own paper, complete the homework assignment.

### 14.2 Systems with Three Variables

14.2a Simple

To solve systems with three variables we must $\qquad$ the $\qquad$ variable $\qquad$
This will give us equations with variables we can then solve for!

## Example 1:

$$
\begin{gathered}
3 x-3 y+5 z=16 \\
2 x-6 y-5 z=35 \\
-5 x-12 y+5 z=28
\end{gathered}
$$

## Example 2:

$$
\begin{gathered}
-x+2 y+4 z=-20 \\
-2 x-2 y-3 z=5 \\
4 x-2 y-2 z=26
\end{gathered}
$$

## 14.2b Multiply to Eliminate

To eliminate a variable, we may have to $\qquad$ one or more equations to get $\qquad$

## Example 1:

$$
\begin{gathered}
-2 x-2 y+3 z=-6 \\
3 x-3 y-2 z=-17 \\
5 x-4 y+5 z=11
\end{gathered}
$$

Q1:

You have completed the videos for 14.2 Systems with Three Variables. On your own paper, complete the homework assignment.

### 14.3 Applications of Systems

## 14.3a Value Comparison

Define the $\qquad$
Make an equation for the $\qquad$
Make an equation for the $\qquad$

## Example 1:

Brian has twice as many dimes as quarters. If the value of the coins is $\$ 4.95$, how many of each does he have?

## Example 2:

A child has three more nickels than dimes in her piggybank. If she has $\$ 1.95$ in her bank, how many of each does she have?

Q1:

## 14.3b Value with Total

Define the $\qquad$
Make an equation for the $\qquad$
Make an equation for the $\qquad$

## Example 1:

Scott has $\$ 2.25$ in his pocket made up of quarters and dimes. If there are 12 coins, how many of each coin does he have?

## Example 2:

If 105 people attended a concert and tickets for adults cost $\$ 2.50$ while tickets for children cost
$\$ 1.75$ and total receipts for the concert were \$228, how many children and how many adults went to the concert?

Define the $\qquad$
Make an equation for the $\qquad$
Make an equation for the $\qquad$
Beware: When using a percent, we must $\qquad$

## Example 1:

Sophia invested \$1900 in one account and \$1500 in another account that paid $3 \%$ higher interest rate. After one year she had earned \$113 in interest. At what rates did she invest?

## Example 2:

Carlos invested \$2500 in one account and \$1000 in another which paid $4 \%$ lower interest. At the end of a year he had earned $\$ 345$ in interest. At what rates did he invest?

Define the $\qquad$
Make an equation for the $\qquad$
Make an equation for the $\qquad$
Beware: When using a percent, we must

## Example 1:

A woman invests $\$ 4600$ in two different accounts. The first paid $13 \%$, the second paid $12 \%$ interest. At the end of the first year she had earned \$586 in interest. How much was in each account?

## Example 2:

A bank loaned out $\$ 4900$ to two different companies. The first loan had a $4 \%$ interest rate; the second had a $13 \%$ interest rate. At the end of the first year the loan had accrued $\$ 421$ in interest. How much was loaned at each rate?

Q1:
Q2:

You have completed the videos for 14.3 Application of Systems - Value problems. On your own paper, complete the homework assignment.

### 14.4 Applications of Systems

14.4a Mixture with Starting Amount

Define the $\qquad$
Make an equation for the $\qquad$
Make an equation for the $\qquad$

## Example 1:

A store owner wants to mix chocolate and nuts to make a new candy. How many pounds of chocolate which costs $\$ 1.50$ per pound should be mixed with 40 pounds of nuts that cost $\$ 3.00$ per pound to make a mixture worth $\$ 2.50$ per pound?

## Example 2:

You need a 55\% alcohol solution. On hand, you have 600 mL of $10 \%$ alcohol mixture. You also have a $95 \%$ alcohol mixture. How much of the $95 \%$ mixture should you add to obtain your desired solution?

Define the $\qquad$
Make an equation for the $\qquad$
Make an equation for the $\qquad$

## Example 1:

A chemist needs to create 100 mL of a $38 \%$ acid solution. On hand she has a $20 \%$ acid solution and a $50 \%$ acid solution. How many mL of each should she use?

## Example 2:

A coffee distributor needs to mix a coffee blend that normally sells for $\$ 8.90$ per pound with another coffee blend that normally sells for \$11.16 per pound, how many pounds of each kind of coffee should they mix if the distributer needs 50 pounds of the new mix to sell for $\$ 9.85$ ?

Pure water is $\qquad$ alcohol

Pure alcohol or acid is $\qquad$ alcohol or acid

## Example 1:

You need a 55\% alcohol solution. On hand, you have a 385 mL of a $70 \%$ alcohol mixture. How much pure water will you need to add to obtain the desired solution?

## Example 2:

You need a 30\% alcohol solution. You have on hand 210 mL of a $10 \%$ alcohol solution. How much pure alcohol do you need to add to obtain the desired solution?

You have completed the videos for 14.4 Applications of Systems - Mixture problems. On your own paper, complete the homework assignment.

Congratulations! You made it through the material for Unit 14: Systems of Equations. It is time to prepare for your exam. On a separate sheet of paper, complete the practice test. Once you have completed the practice test, ask your instructor to take the test. Good luck!

## Unit 15: Radicals

To work through the unit, you should:

1. Watch a video, as you watch, fill out the workbook (top and example sections).
2. Complete Q1 and Q2 in WAMAP, put your work in the right column of the page.
3. Repeat \#1 and \#2 with each page until you reach the .
4. Complete the homework assignment on your own paper.
5. Repeat \#1 thru \#4 until you reach the end of the unit.
6. Complete the review/practice test on your own paper.
7. Take the unit exam.

### 15.1 Simplify Radicals

## 15.1a Variables

Radical: $\sqrt[n]{a}=b$ where $\qquad$ . The $n$ is called the $\qquad$ .

Square Root: $\sqrt{a}=b$ where $\qquad$ . The index on a square root is always $\qquad$
Radicals divide the $\qquad$ by the $\qquad$
The whole number is how many "things" $\qquad$ and the remainder is how many "things" $\qquad$

## Example 1:

$$
\sqrt{a^{3}}
$$

## Example 2:

Q1:

Q2:
$\sqrt[4]{b^{19}}$

## 15.1b Several Variables

Work with $\qquad$ variable at a time

Example 1:

$$
\sqrt{a^{5} b^{8} c^{15}}
$$

## Example 2:

Q1:

Q2:

$$
\sqrt[4]{a^{13} b^{23} c^{10} d^{3} e^{36}}
$$

## 15.1c Using Prime Factorization

Prime Factorization:
To find a prime factorization we $\qquad$ by $\qquad$
A few prime numbers:
Roots of numbers are difficult, find the $\qquad$ so that we can divide the $\qquad$ by the $\qquad$

## Example 1:

$$
\sqrt[3]{750}
$$

## Example 2:

Q1:

Q2:

$$
9 \sqrt{250 x^{4} y z^{5}}
$$

We can only pull $\qquad$ (separated by $\qquad$ ) out of a radical If we have $\qquad$ (separated by $\qquad$ or $\qquad$ ) we must $\qquad$ first!

## Example 1:

$$
\sqrt{100 x^{2}-16 x^{4}}
$$

## Example 2:

$$
\sqrt[3]{216 x^{6}-27 x^{9}}
$$

Q1:

Q2:

### 15.2 Add, Subtract and Multiply Radicals

15.2a Add Like Radicals

Simplify: $2 x-5 y+4 x+2 y$
Simplify: $2 \sqrt{3}-5 \sqrt{7}+4 \sqrt{3}+2 \sqrt{7}$
When adding and subtracting radicals we can

Example 1:

$$
-4 \sqrt{6}+2 \sqrt{11}+\sqrt{11}-5 \sqrt{6}
$$

## Example 2:

Q1:

Q2:

$$
\sqrt[3]{5}+3 \sqrt{5}-8 \sqrt[3]{5}+2 \sqrt{5}
$$

> 15.2b Add with Simplifying

Before adding radicals together $\qquad$

Example 1:

$$
5 \sqrt{50 x}+5 \sqrt{27}-3 \sqrt{2 x}-2 \sqrt{108}
$$

## Example 2:

$\sqrt[3]{81 x^{3} y}-3 y \sqrt[3]{32 x^{2}}+x \sqrt[3]{24 y}-\sqrt[3]{500 x^{2} y^{3}}$

Q1:

Q2:
az

## 15.2c Multiply Monomial Radical Expressions

Product Rule: $a \sqrt[n]{b} \llbracket \sqrt[n]{d}=$
Always be sure your final answer is $\qquad$

## Example 1:

$$
4 \sqrt{6} \square 2 \sqrt{15}
$$

## Example 2:

$-3 \sqrt[4]{8} \square \sqrt[4]{10}$

Recall: $a(b+c)=$
Always be sure your final answer is

## Example 1:

$$
5 \sqrt{10}(2 \sqrt{6}-3 \sqrt{15})
$$

## Example 2:

Q1:

Q2:

$$
7 \sqrt{3}(\sqrt{6}+9)
$$

## 15.2e Multiply Binomial Radical Expressions

Recall: $(a+b)(c+d)=$
Always be sure your final answer is

## Example 1:

$$
(3 \sqrt{7}-2 \sqrt{5})(\sqrt{7}+6 \sqrt{5})
$$

Example 2:

$$
(2 \sqrt[3]{9}+5)(4 \sqrt[3]{3}-1)
$$

## 15.2f Square Binomial Radical Expression

Recall: $(a+b)^{2}=$
Always be sure your final answer is $\qquad$

## Example 1:

$$
(\sqrt{6}-\sqrt{2})^{2}
$$

## Example 2:

Q1:

Q2:

$$
(2+3 \sqrt{7})^{2}
$$

## 15.2 g . Multiply Conjugates

Recall: $(a+b)(a-b)=$
Always be sure your final answer is $\qquad$

## Example 1:

$$
(4+2 \sqrt{7})(4-2 \sqrt{7})
$$

Example 2:

$$
(2 \sqrt{3}-\sqrt{6})(2 \sqrt{3}+\sqrt{6})
$$

Q1:

Q2:

### 15.3 Rationalize Denominator

## 15.3a Simplifying with Radicals

Expression with radicals: Always $\qquad$ the $\qquad$ first

Before $\qquad$ with fractions, be sure to $\qquad$ first

Example 1:

## Example 2:

$$
\frac{15+\sqrt{175}}{10}
$$

Q1:

Q2:

$$
\frac{8-\sqrt{48}}{6}
$$

15.3b Quotient Rule

Quotient Rule: $\sqrt{\frac{a}{b}}=$
It may be helpful to reduce the $\qquad$ first and the $\qquad$ second

Example 1:

$$
\frac{\sqrt{48}}{\sqrt{150}}
$$

## Example 2:

$$
\sqrt{\frac{225 x^{7}}{20 x^{3}}}
$$

15.3c Rationalize Monomial Roots in the Denominator

Rationalize Denominators: Never leave a $\qquad$ in the $\qquad$
To clear radicals: $\qquad$ by extra needed factors in denominator (same in numerator!)

It may be helpful to $\qquad$ first

Hint: $\qquad$ numbers!

## Example 1:

$$
\frac{5}{\sqrt[7]{b^{2}}}
$$

## Example 2:

Q1:

Q2:

$$
\sqrt[3]{\frac{7}{9 a^{2} b}}
$$

What does not work: $\frac{1}{2+\sqrt{3}}=$
Recall: $(2+\sqrt{3})(\quad)=$
Multiply by the $\qquad$

## Example 1:

$$
\frac{6}{5-\sqrt{3}}
$$

Q1:

Q2:

$$
\frac{3-5 \sqrt{2}}{4+2 \sqrt{2}}
$$

### 15.4 Rational Exponents

15.4a Convert

If we divide the exponent by the index, then $\sqrt[n]{a^{m}}=$
The index is the $\qquad$

## Example 1:

Write as an exponent: $\sqrt[7]{m^{5}}$

## Example 2:

$$
\text { Write as a radical: }(a b)^{2 / 3}
$$

## Example 3:

Write as a radical: $x^{-4 / 5}$

## Example 4:

Write as an exponent: $\frac{1}{(\sqrt[3]{5 x})^{2}}$

Q1:

Q2:

Q3:

Q4:

## 15.4b Evaluate

To evaluate a rational exponent $\qquad$ to a $\qquad$

## Example 1:

$$
\text { Evaluate: } 32^{2 / 5}
$$

## Example 2:

Evaluate: $27^{-4 / 3}$

## 15.4c Simplify

Recall Exponent Properties
$a^{m} a^{n}=$
$\left(\frac{a}{b}\right)^{m}=$
$\frac{a^{m}}{a^{n}}=$
$\left(a^{m}\right)^{n}=$
$(a b)^{m}=$
$a^{0}=$
$\left(\frac{a}{b}\right)^{-m}=$

$$
a^{-m}=
$$

$$
\frac{1}{a^{-m}}=
$$

To Simplify:

## Example 1:

$$
\frac{x^{4 / 3} y^{2 / 7} x^{5 / 4} y^{3 / 7}}{x^{1 / 2} y^{6 / 7}}
$$

## Example 2:

$$
\left(\frac{256 x^{3 / 2} y^{-1 / 3}}{x^{1 / 4} y^{3 / 2} x^{-5 / 2}}\right)^{-1 / 8}
$$

Q1:
Q2: homework assignment

### 15.5 Radicals of Mixed Index <br> 15.5a Reduce Index

Using rational exponents: $\sqrt[8]{x^{6} y^{2}}=$

To reduce the index $\qquad$ the $\qquad$ and the $\qquad$ by the $\qquad$ Without using rational exponents: $\sqrt[8]{x^{6} y^{2}}=$

Hint: $\qquad$ any numbers

## Example 1:

$$
\sqrt[15]{x^{3} y^{9} z^{6}}
$$

## Example 2:

Q1:

Q2:

$$
\sqrt[25]{32 a^{10} b^{5} c^{20}}
$$

## 15.5b Multiply Mixed Index

Using rational exponents: $\sqrt[3]{a^{2} b} \sqsubset \sqrt[4]{a b^{2}}=$

Get a $\qquad$ by $\qquad$ the $\qquad$ and $\qquad$
Without using rational exponents: $\sqrt[3]{a^{2} b} \square \sqrt[4]{a b^{2}}=$
Hint: $\qquad$ any numbers

Always be sure your final answer is $\qquad$

## Example 1:

$$
\sqrt[4]{m^{3} n^{2} p} \sqrt[6]{m n^{2} p^{3}}
$$

Example 2:

Q1:

Q2:

$$
\sqrt[3]{4 x^{2} y} \square \sqrt[5]{8 x^{4} y^{2}}
$$

Division with mixed index - get a $\qquad$
Hint: $\qquad$ any numbers

May have to $\qquad$ the denominator (cannot be under a $\qquad$ and under a $\qquad$ _)

## Example 1:

$$
\frac{\sqrt{a b^{3}}}{\sqrt[3]{a b^{2}}}
$$

## Example 2:

Q1:

Q2:

$$
\frac{\sqrt[4]{2 x^{3} y^{2}}}{\sqrt[6]{32 y^{4}}}
$$ homework assignment.

### 15.6 Complex Numbers <br> 15.6a Square Roots of Negatives

Define: $\sqrt{-1}=\quad$ and therefore $i^{2}=$ Now we can calculate $\sqrt{-25}=$

Expressions with radicals: Always $\qquad$ the $\qquad$ first

## Example 1:

$$
\sqrt{-45}
$$

## Example 2:

Q2:

$$
\sqrt{-6} \square \sqrt{-10}
$$

Before $\qquad$ with fractions, be sure to $\qquad$ first

Example 1:

$$
\frac{15+\sqrt{-300}}{5}
$$

## Example 2:

$$
\frac{20+\sqrt{-80}}{8}
$$

$i$ works just like $\qquad$
This means we can $\qquad$

Example 1:

$$
(5-3 i)+(6+i)
$$

## Example 2:

$$
(-5-2 i)-(3-6 i)
$$

Q1:

Q2:
$i^{0}=$
$i^{1}=$
$i^{2}=$
$i^{3}=$
$\qquad$ the exponent by $\qquad$ and use the $\qquad$

## Example 1:

$i^{223}$

Q1:

Q2:
$i^{96}$

## 15.6e Multiply

$i$ works just like $\qquad$
Remember $i^{2}=$

## Example 1:

$$
(-3 i)(6 i)
$$

Example 2:

$$
2 i(5-2 i)
$$

Example 3:

$$
(4-3 i)(2-5 i)
$$

## Example 4:

$$
(3+2 i)^{2}
$$

Q1:

Q2:

Q3:

Q4:

If $i=$ then we can rationalize it by just multiplying by $\qquad$

Example 1:

$$
\frac{5+3 i}{4 i}
$$

Example 2:

$$
\frac{2-i}{-3 i}
$$

Q1:

Q2:
Q2:

Similar to other radicals we can rationalize a binomial by multiplying by the $\qquad$ $(a+b i)(a-b i)=$

## Example 1:

$$
\frac{4 i}{2-5 i}
$$

Example 2:

$$
\frac{4-2 i}{3+5 i}
$$

Q1:

Q2:

### 15.7 Complete the Square

15.7a Find c
$a^{2}+2 a b+b^{2}$ is easily factored to $\qquad$
To make $x^{2}+b x+c$ a perfect square, $c=$

## Example 1:

Find $c$ and factor the perfect square:

$$
x^{2}+10 x+c
$$

## Example 2:

Find $c$ and factor the perfect square

$$
x^{2}-7 x+c
$$

## Example 3:

Find $c$ and factor the perfect square:

$$
x^{2}-\frac{3}{7} x+c
$$

## Example 4:

Find $c$ and factor the perfect square:

$$
x^{2}+\frac{6}{5} x+c
$$

Q4:
Q1:

Q2:

Q3:

## 15.7b Rational Solutions

If $x^{2}=9$ then there are $\qquad$ solutions for $x$, $\qquad$ and $\qquad$ . We can write this as $\qquad$
To complete the square on $a x^{2}+b x+c=0$

1. Separate $\qquad$ and $\qquad$
2. Divide by $\qquad$ (everything)
3. Find the $\qquad$ and $\qquad$ to $\qquad$

## Example 1:

$$
x^{2}-x-6=0
$$

## Example 2:

$$
3 x^{2}=15 x-18
$$

Q1:
15.7c Irrational and Complex Solutions

If we can't simplify the $\qquad$ we $\qquad$ what we can.

Example 1:

$$
5 x^{2}-3 x+2=0
$$

## Example 2:

$$
8 x+32=4 x^{2}
$$

Q1:
Q2:

You have completed the videos for 15.7 Complete the Square. On your own paper, complete the homework assignment.

### 15.8 Quadratic Formula

15.8a Finding the Formula

Solve by Completing the Square:

$$
a x^{2}+b x+c=0
$$

(Finding the Formula is useful to know for the test!)

## 15.8b Using the Formula

If $a x^{2}+b x+c=0$ the $x=$

## Example 1:

$$
6 x^{2}+7 x-3=0
$$

## Example 2:

$$
5 x^{2}-x+2=0
$$

## 15.8c Make Equation Equal Zero

Before using the quadratic formula, the equation must equal $\qquad$ and be in $\qquad$
That is the equation should look like:

## Example 1:

$$
2 x^{2}=15-7 x
$$

## Example 2:

$$
3 x^{2}+5 x+2=7
$$

If a term is missing, we use $\qquad$ in the quadratic formula

Example 1:

$$
3 x^{2}+54=0
$$

Example 2:

$$
5 x^{2}=2 x
$$

Q1:

Q2:

## Unit 16: College Algebra Topics

To work through the unit, you should:

1. Watch a video, as you watch, fill out the workbook (top and example sections).
2. Complete Q1 and Q2 in WAMAP, put your work in the right column of the page.
3. Repeat \#1 and \#2 with each page until you reach the .
4. Complete the homework assignment on your own paper.
5. Repeat \#1 thru \#4 until you reach the end of the unit.
6. Complete the review/practice test on your own paper.
7. Take the unit exam.

### 16.1 Multiply and Divide Rational Expressions

## 16.1a Review Multiply and Divide Fractions

To multiply we $\qquad$ common $\qquad$ then multiply $\qquad$
Division is the same, with one extra step at the start: $\qquad$ by the $\qquad$

Example 1:

$$
\frac{6}{35}-\frac{21}{10}
$$

## Example 2:

$\frac{5}{8} \div \frac{10}{3}$

Q1:

Q2:

## 16.1b Multiply or Divide Rational Expressions

To multiply we $\qquad$ common $\qquad$ then multiply $\qquad$
This means we must first $\qquad$
Division is the same, with one extra step at the start: $\qquad$ by the $\qquad$

## Example 1:

$$
\frac{x^{2}+3 x+2}{4 x-12} \sqrt{x^{2}-5 x+6} \frac{x^{2}-4}{}
$$

Example 2:

$$
\frac{3 x^{2}+5 x-2}{x^{2}+3 x+2} \div \frac{6 x^{2}+x-1}{2 x^{3}-6 x^{2}-8 x}
$$

Q1:

Q2:

To divide:
To multiply we $\qquad$ common $\qquad$ then multiply $\qquad$
This means we must first $\qquad$

## Example 1:

$$
\frac{x^{2}+3 x-10}{x^{2}+6 x+5} \div \frac{2 x^{2}-x-3}{2 x^{2}+x-6} \div \frac{8 x+20}{6 x+15}
$$

## Example 2:

Q1:

Q2:

$$
\frac{x^{2}-1}{x^{2}-x-6} \square \frac{2 x^{2}-x-15}{3 x^{2}-x-4} \div \frac{2 x^{2}+3 x-5}{3 x^{2}+2 x-8}
$$

### 16.2 Add and Subtract Rational Expressions

## 16.2a Review LCD/LCM of Numbers with Prime Factorization

Prime Factorization:
To find the LCD/LCM use $\qquad$ factors with $\qquad$ exponents

## Example 1:

Find the LCD/LCM:
20 and 36

## Example 2:

## Q1:

Q2:

Find the LCD/LCM:
18,54 and 81

## 16.2b LCD/LCM of Monomials

To find the LCD/LCM with variables use $\qquad$ factors with $\qquad$ exponents

Example 1:
Find the LCD/LCM:
$5 x^{3} y^{2}$ and $4 x^{2} y^{5}$

## Example 2:

Find the LCD/LCM:
$7 a b^{2} c$ and $3 a^{4} b$

Q1:

Q2:

To find the LCD/LCM with polynomials use $\qquad$ factors with $\qquad$ exponents

This means we must first $\qquad$

## Example 1:

Find the LCD/LCM:

$$
x^{2}+3 x-18 \text { and } x^{2}+4 x-21
$$

## Example 2:

## Q1:

Q2:

Find the LCD/LCM:
$x^{2}-10 x+25$ and $x^{2}-x-20$

## 16.2d Review Adding and Subtracting Fractions

To add or subtract we $\qquad$ the denominators by $\qquad$ by the missing $\qquad$

Example 1:

$$
\frac{5}{21}+\frac{7}{15}
$$

## Example 2:

Q1:

Q2:

$$
\frac{8}{14}-\frac{3}{10}
$$

## 16.2e Add and Subtract with Common Denominator

Add the $\qquad$ and keep the $\qquad$
When subtracting we will first $\qquad$ the negative

Don't forget to $\qquad$

## Example 1:

$$
\frac{x^{2}+4 x}{x^{2}-2 x-15}+\frac{x+6}{x^{2}-2 x-15}
$$

## Example 2:

$$
\frac{x^{2}+2 x}{2 x^{2}-9 x-5}-\frac{6 x+5}{2 x^{2}-9 x-5}
$$

To add or subtract we $\qquad$ the denominators by $\qquad$ by the missing $\qquad$ This means we must first $\qquad$ the denominators

Example 1:

$$
\frac{2 x}{x^{2}-9}+\frac{5}{x^{2}+x-6}
$$

## Example 2:

$$
\frac{2 x+7}{x^{2}-2 x-3}-\frac{3 x-2}{x^{2}+6 x+5}
$$

Q1:

Q2:
.

### 16.3 Compound Fractions

## 16.3a Numbers

Compound/Complex Fractions:
Clear $\qquad$ by multiplying each $\qquad$ by the $\qquad$ of everything

## Example 1:

$$
\frac{\frac{3}{4}+\frac{5}{6}}{\frac{1}{2}-\frac{4}{3}}
$$

## Example 2:

Q2:

$$
\frac{\frac{1}{2}+2}{1+\frac{9}{4}}
$$

## 16.3b Monomials

Recall: To find the LCD with variables, use the $\qquad$ exponents

Be sure to check for $\qquad$ by $\qquad$ the numerator and denominator.

## Example 1:

$$
\frac{1-\frac{9}{x^{2}}}{\frac{1}{x}+\frac{3}{x^{2}}}
$$

Example 2:

$$
\frac{\frac{1}{y^{3}}-\frac{1}{x^{3}}}{\frac{1}{x^{2} y^{3}}-\frac{1}{x^{3} y^{2}}}
$$

## Q1:

Q2:

Recall: To find the LCD with variables, use the $\qquad$ exponents.

Be sure to check for $\qquad$ by $\qquad$ the numerator and denominator.

## Example 1:

## Example 2:

$$
\frac{\frac{5}{x-2}}{3+\frac{2}{x-2}}
$$

Q1:

Q2:
$\frac{\frac{x}{x-9}+\frac{5}{x+9}}{\frac{x}{x+9}-\frac{5}{x-9}}$

## 16.3d Negative Exponents

Recall: $5 x^{-3}=$
If there is any $\qquad$ or $\qquad$ we can't just $\qquad$ terms. Instead make $\qquad$

Example 1:

## Example 2:

$$
\frac{1+10 x^{-1}+25 x^{-2}}{1-25 x^{-2}}
$$

Q2:

$$
\frac{8 b^{-3}+27 a^{-3}}{4 a^{-1} b^{-3}-6 a^{-2} b^{-2}+9 a^{-3} b^{-1}}
$$

Q1:

2: homework assignment.

### 16.4 Rational Equations

16.4a Clear Denominator

Recall: $\frac{3}{4} x-\frac{1}{2}=\frac{5}{6}$

Clear fractions by multiplying each $\qquad$ by the $\qquad$

## Example 1:

## Q1:

$$
\frac{5}{x}=\frac{3}{7 x}-4
$$

## Example 2:

Q2:

$$
\frac{4}{x+5}+x=\frac{-2}{x+5}
$$

## 16.4b Factoring Denominator

To identify all the factors in the $\qquad$ we may have to $\qquad$ the $\qquad$

Example 1:

$$
\frac{x}{x-6}+\frac{1}{x-7}=\frac{-3 x-8}{x^{2}-13 x+42}
$$

## Example 2:

Q1:

Q2:

$$
\frac{2}{x+3}-\frac{9 x}{x^{2}-9}=\frac{1}{x-3}
$$

Because we are working with fractions, the $\qquad$ cannot be $\qquad$

Example 1:

## Example 2:

$$
\frac{x}{x-8}-\frac{2}{x-4}=\frac{-3 x+56}{x^{2}-12 x+32}
$$

$$
\frac{x}{x-2}+\frac{2}{x-4}=\frac{4 x-12}{x^{2}-6 x+8}
$$

Q1:

Q2:

### 16.5 Equations with Radicals <br> 16.5a Odd Roots

The opposite of taking a root is to do an $\qquad$
$\sqrt[3]{x}=4$ then $x=$
(Note: This only works for an $\qquad$ index)

## Example 1:

$$
\sqrt[3]{2 x-5}=6
$$

Example 2:

$$
\sqrt[5]{4 x-7}=2
$$

## 16.5b Even Roots

The opposite of taking a root is to do an $\qquad$
With even roots we must $\qquad$ the answer in the original equation! (called $\qquad$ _)

Recall: $(a+b)^{2}=$

## Example 1:

$$
x=\sqrt{5 x+24}
$$

## Example 2:

$$
\sqrt{40-3 x}=2 x-5
$$

Q1:

Q2:

## 16.5c Isolate Radical

IMPORTANT: Before we can clear a radical it must first be $\qquad$

Example 1:

$$
4+2 \sqrt{2 x-1}=2 x
$$

## Example 2:

$$
2 \sqrt{5 x+1}-2=2 x
$$

Q1:
Q2:

You have completed the videos for 16.5 Equations with Radicals. On your own paper, complete the homework assignment.

### 16.6 Equations with Exponents

## 16.6a Odd Exponents

The opposite of taking an exponent is to do a $\qquad$ If $x^{3}=8$, then $x=$
(Note: This only works for an $\qquad$ exponent)

Example 1:

$$
(3 x+5)^{5}=32
$$

Example 2:

$$
(2 x-1)^{3}=64
$$

## 16.6b Even Exponents

Consider: $(5)^{2}=\quad$ and $(-5)^{2}=$
When we clear an even exponent, we have $\qquad$

Example 1:

$$
(5 x-1)^{2}=49
$$

## Example 2:

Q1:

Q2:

$$
(3 x+2)^{4}=81
$$

## 16.6c Isolate Exponent

IMPORTANT: Before we can clear an exponent, it must first be $\qquad$

Example 1:

$$
4-2(2 x+1)^{2}=-46
$$

## Example 2:

$$
5(3 x-2)^{2}+6=46
$$

Q1:

Q2:

To multiply to one: $\frac{a}{b}(\square)=1$
We clear a rational exponent by using a $\qquad$
Recall $a^{m / n}=$
Recall: Check if original rational exponent has $\qquad$
Recall: Two solutions if original rational exponent has $\qquad$

Example 1:

$$
(3 x-6)^{3 / 2}=64
$$

## Example 2:

$$
(5 x+1)^{4 / 5}=16
$$

Q1:

Q2:

### 16.7 Rectangle Problems

## 16.7a Area Problems

Area of a rectangle:
To help visualize the rectangle,
There are three ways to solve any quadratic equation
1.
2.
3.

## Example 1:

The length of a rectangle is 2 ft longer than the width. The area of the rectangle is $48 \mathrm{ft}^{2}$. What are the dimensions of the rectangle?

## Example 2:

The area of a rectangle is $72 \mathrm{~cm}^{2}$. If the width is 6 cm less than the length, what are the dimensions of the rectangle?

Q1:

Q2:

## 16.7b Perimeter Problems

Perimeter of a rectangle:
Tip: Solve the $\qquad$ equation for a variable and $\qquad$ in the $\qquad$ equation.

## Example 1:

The area of a rectangle is $54 \mathrm{~m}^{2}$. If the perimeter is 30 meters, what are the dimensions of the rectangle?

## Example 2:

The perimeter of a rectangle is 22 inches. If the area of the same rectangle is $24 \mathrm{in}^{2}$, what are the dimensions?

Q1:

Q2:

## 16.7c Bigger

We may have to draw $\qquad$ rectangles

Multiply/Add to the $\qquad$ to make it equal the $\qquad$ rectangle

## Example 1:

Each side of a square is decreased 6 inches. When this happens, the area of the larger square is 16 times the area of the smaller square. How many inches is the side of the original square?

## Example 2:

The length of a rectangle is 9 feet longer than it is wide. If each side is increased 9 feet, then the area is multiplied by 3 . What are the dimensions of the original rectangle?

Q1:

Q2:

## 16.7d Frames

To help visualize the frame $\qquad$
Remember the frame is on the $\qquad$ and $\qquad$ also the $\qquad$ and $\qquad$

## Example 1:

A frame measures 13 inches by 10 inches and is of uniform width. If the area of the picture inside is 54 square inches, what is the width of the frame?

## Example 2:

An 8-inch by 12 -inch drawing has a frame of uniform width around it. The area of the frame is equal to the area of the picture. What is the width of the frame?

Q1:

Q2:

## 16.7e Percent of a Field

Clearly identify the area of the $\qquad$ and $\qquad$ rectangles!

Be careful with $\qquad$ , is it talking about the $\qquad$ , $\qquad$ , or $\qquad$ ?

## Example 1:

A man mows his 40 ft by 50 ft rectangular lawn in a spiral pattern starting from the outside edge. By noon he is $90 \%$ done. How wide of a strip has he cut around the outside edge?

## Example 2:

A woman has a 50 ft by 25 ft rectangular field that she wants to increase by $68 \%$ by cultivating a strip of uniform width around the current field. How wide of a strip should she cultivate?

## Q1:

Q2:

### 16.8 Work Problems

16.8a One Unknown Time

Adam does a job in 4 hours. Each hour he does $\qquad$ of the job.

Betty does a job in 12 hours. Each hour she does $\qquad$ of the job.

Together, each hour they do $\qquad$ of the job

This means together it would take them $\qquad$ hours to do the entire job.

Work equation:

## Example 1:

Catherine can paint a house in 15 hours. Dan can paint it in 30 hours. How long will it take them working together?

## Example 2:

Even can clean a room in 3 hours. If his sister Faith helps, it takes them $2 \frac{2}{5}$ hours. How long will it take Faith working alone?

## Q1:

Q2:

Be sure to clearly identify who is the $\qquad$

## Example 1:

Tony does a job in 16 hours less time than Marissa, and they can do it together in 15 hours. How long will it take each to do the job alone?

## Example 2:

Alex can complete his project in 21 hours less than Hillary. If they work together it can get done in 10 hours. How long does it take each working alone?

## Q1:

Q2: homework assignment.

### 16.9 Distance and Revenue Problems <br> 16.9a Simultaneous Products

Simultaneous product: $\qquad$ equations with $\qquad$ variables that are $\qquad$ To solve: $\qquad$ both by the same $\qquad$ Then $\qquad$

Example 1:

$$
\begin{gathered}
x y=72 \\
(x-5)(y+2)=56
\end{gathered}
$$

Q1:

## 16.9b Revenue

Revenue Equation:
Beware: Profit =
To solve: Divide by what we $\qquad$

## Example 1:

A group of college students bought a couch for $\$ 80$. However, five of them failed to pay their share so the others had to each pay $\$ 8$ more. How many students were in the original group?

## Example 2:

A merchant bought several pieces of silk for $\$ 70$. He sold all but two of them at a profit of $\$ 4$ per piece. His total profit was $\$ 18$. How many pieces did he originally purchase?

Q1:
Q2:

Distance Equation:
To solve: Divide by what we

## Example 1:

A man rode his bike to a park 60 miles away. On the return trip he went 2 mph slower which made the trip take 1 hour longer. How fast did he ride to the park?

## Example 2:

After driving through a construction zone for 45 miles, a woman realized that if she had just driven 6 mph faster, she would have arrived 2 hours sooner. How fast did she drive?

Downwind/stream:
Upwind/stream:

## Example 1:

Zoe rows a boat downstream for 80 miles. The return trip upstream took 12 hours longer. If the current flows at 3 mph , how fast does Zoe row in still water?

## Example 2:

Darius flies a plane against a headwind for 5084 miles. The return trip with the wind took 20 hours less time. If the wind speed is 10 mph , how fast does Darius fly the plane when there is no wind?

You have completed the videos for 16.9 Distance and Revenue Problems. On your own paper, complete the homework assignment.

Congratulations! You made it through the material for Unit 16: College Algebra Topics. It is time to prepare for your exam. On a separate sheet of paper, complete the practice test. Once you have completed the practice test, ask your instructor to take the test. Good luck!

## Unit 17: Functions

To work through the unit, you should:

1. Watch a video, as you watch, fill out the workbook (top and example sections).
2. Complete Q 1 and Q 2 in WAMAP, put your work in the right column of the page.
3. Repeat \#1 and \#2 with each page until you reach the .
4. Complete the homework assignment on your own paper.
5. Repeat \#1 thru \#4 until you reach the end of the unit.
6. Complete the review/practice test on your own paper.
7. Take the unit exam.

### 17.1 Evaluate Functions

## 17.1a Functions

Function:
If it is a function, we often write $\qquad$ which is read $\qquad$
A graph is a function if it passes the $\qquad$ , or each $\qquad$ has at most one $\qquad$

## Example 1:

Is the graph a function?


## Example 2:

Is the graph a function?


Q1:


Q2:


## 17.1b Function Notation

Function notation:
What is inside of the function $\qquad$ the $\qquad$

## Example 1:

$$
f(x)=-x^{2}+2 x-5
$$

Find $f(3)$

Example 2:

$$
g(x)=\sqrt{2 x+5}
$$

Find $g(20)$

## 17.1c Evaluate Function at an Expression

When replacing a variable, we always use $\qquad$
What is inside of the function $\qquad$ the $\qquad$

## Example 1:

$$
\begin{aligned}
& f(x)=\sqrt{2 x}+3 x \\
& \text { Find } f\left(8 x^{2}\right)
\end{aligned}
$$

## Example 2:

$$
p(n)=n^{2}-2 n+5
$$

Find $p(n-3)$

## Q1:

Q2:

## 17.1d Domain

Domain:
Fractions:
Even Radicals:

## Example 1:

Find the domain:

$$
f(x)=3 \sqrt[4]{2 x-6}+4
$$

## Example 2:

Find the domain:

$$
g(x)=3|2 x+7|^{2}-4
$$

## Example 3:

Find the domain:
$h(x)=\frac{x-1}{x^{2}-x-2}$

## Q1:

Q2:

### 17.2 Operations on Functions

17.2a Add Functions

Add Functions: $(f+g)(x)=$
With a number we will $\qquad$ both, then $\qquad$ the results

With a variable we will $\qquad$ the two functions $\qquad$ Use $\qquad$ !

## Example 1:

$$
\begin{gathered}
f(x)=x-4 \\
g(x)=x^{2}-6 x+8
\end{gathered}
$$

Find $(f+g)(-2)$

Example 2:

$$
\begin{gathered}
f(x)=x^{2}-5 x \\
g(x)=x-5
\end{gathered}
$$

Find $(f+g)(x)$

Q1:

Q2:

## 17.2b Subtract Functions

Subtract Functions: $(f-g)(x)=$
With a number we will $\qquad$ both, then $\qquad$ the results

With a variable we will $\qquad$ the two functions $\qquad$ . Use $\qquad$

## Example 1:

$$
\begin{gathered}
f(x)=x-4 \\
g(x)=x^{2}-6 x+8
\end{gathered}
$$

Find $(f-g)(-2)$

## Example 2:

Q1:

Q2:

$$
\begin{gathered}
f(x)=x^{2}-5 x \\
g(x)=x-5
\end{gathered}
$$

Find $(f-g)(x)$

## 17.2c Multiply Functions

Multiply Functions: $(f \square g)(x)=$
With a number we will $\qquad$ both, then $\qquad$ the results

With a variable we will $\qquad$ the two functions $\qquad$ Use $\qquad$ !

## Example 1:

$$
\begin{gathered}
f(x)=x-4 \\
g(x)=x^{2}-6 x+8
\end{gathered}
$$

Find $(f \square g)(-2)$

## Example 2:

Q1:

Q2:

$$
\begin{gathered}
f(x)=x^{2}-5 x \\
g(x)=x-5
\end{gathered}
$$

Find $(f \sqcap g)(x)$

## 17.2d Divide Functions

Divide Functions: $\left(\frac{f}{g}\right)(x)=$
With a number we will $\qquad$ both, then $\qquad$ the results With a variable we will $\qquad$ the two functions $\qquad$ . Use $\qquad$ Beware of $\qquad$ of fractions, the $\qquad$ cannot be $\qquad$

## Example 1:

$$
\begin{gathered}
f(x)=x-4 \\
g(x)=x^{2}-6 x+8
\end{gathered}
$$

Find $\left(\frac{f}{g}\right)(-2)$

## Example 2:

Q1:

Q2:

$$
\begin{gathered}
f(x)=x^{2}-5 x \\
g(x)=x-5
\end{gathered}
$$

Find $\left(\frac{f}{g}\right)(x)$

## 17.2e Composition of Functions

Composition of Functions:
$(f \circ g)(x)=$
With numbers, $\qquad$ the $\qquad$ and put $\qquad$ in $\qquad$
With a variable, put the $\qquad$ in for the $\qquad$ in the $\qquad$

## Example 1:

$$
\begin{gathered}
f(x)=\sqrt{x+6} \\
g(x)=x+3
\end{gathered}
$$

$(f \circ g)(7)=$
$g[f(7)]=$

## Example 2:

$$
\begin{gathered}
p(x)=x^{2}+2 x \\
r(x)=x+3
\end{gathered}
$$

$(p \circ r)(x)=$

$$
r[p(n)]=
$$

17.2f Compose a Function with Itself

A function can be composed with $\qquad$

Example 1:

$$
f(x)=2 x-4
$$

Find $(f \circ f)(-2)$

Example 2:
Q1:

Q2:

$$
g(x)=x^{2}-3 x
$$

Find $g[g(x)]$

If we are composing several functions, start in the $\qquad$ and work $\qquad$

Example 1:

$$
\begin{gathered}
f(x)=x+2 \\
g(x)=x^{2}-5 \\
h(x)=\sqrt{3 x}
\end{gathered}
$$

Find $(f \circ g \circ h)(2)$

## Example 2:

$$
\begin{gathered}
f(x)=x+2 \\
g(x)=x^{2}-5 \\
h(x)=\sqrt{3 x}
\end{gathered}
$$

Find $(f \circ g \circ h)(a)$

Q1:
Q2:

You have completed the videos for 17.2 Operations on Functions. On your own paper, complete the homework assignment.

### 17.3 Inverse Functions <br> 17.3a Show Functions are Inverses

Inverse Function:
To test if functions are inverses, calculate $\qquad$ and $\qquad$ , the answer to both should be $\qquad$

## Example 1:

Are they inverses?

$$
\begin{aligned}
& f(x)=3 x-8 \\
& g(x)=\frac{x}{3}+8
\end{aligned}
$$

## Example 2:

Are they inverses?

$$
\begin{aligned}
& f(x)=\frac{5}{x-3}+6 \\
& g(x)=\frac{5}{x-6}+3
\end{aligned}
$$

Q1:
Q2:
17.3b Finding an Inverse Function

To find an inverse function $\qquad$ the $\qquad$ and $\qquad$ , then solve for $\qquad$ . (the $\qquad$ is the $y!$ )

## Example 1:

Find the inverse:

$$
h(x)=\frac{-3}{x-1}-2
$$

## Example 2:

Find the inverse:

$$
g(x)=5 \sqrt[3]{x-6}+4
$$

Q1:
Q2:

## 17.3c Inverse of Rational Functions

Clear fractions by $\qquad$
Put the terms with $\qquad$ on one side and $\qquad$ on the other side

Factor out the $\qquad$ and $\qquad$ to get it alone

## Example 1:

Find the inverse:

$$
f(x)=\frac{2 x-5}{x+3}
$$

## Example 2:

Find the inverse:

$$
g(x)=\frac{5 x+1}{2 x-5}
$$

Q1:
Q2:

STOP
You have completed the videos for 17.3 Inverse Functions. On your own paper, complete the homework assignment.

### 17.4 Graphs of Quadratic Functions <br> 17.4a Key Points

Quadratic Graph:
Key points:


## Example 1:

Graph the function:

$$
f(x)=x^{2}-2 x-3
$$



## Example 2:

Graph the function

$$
f(x)=-3 x^{2}+12 x-9
$$



You have completed the videos for 17.4 Graphs of Quadratic Functions. On your own paper, complete the homework assignment.

Congratulations! You made it through the material for Unit 17: Functions. It is time to prepare for

## Unit 18: <br> Proficiency Exam \#3

To work through this unit, you should:

1. Complete the review/practice tests on your own paper.
2. Take the (two part) unit exam.
